Effect of End Conditions on Free Vibration of Thin Laminated Rectangular Composite Plates with Exact Displacement Functions

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Abstract— This paper presents the effect of end conditions on free vibration of thin rectangular laminated composite plates with exact displacement function. The work aims at obtaining the exact free vibration equation of different end conditions of thin rectangular laminated composite plate which will not depend on assumed shape function rather on the deflected shape of the plates. The equation derived in this paper is based classical plate theory which is widely used for analysis of thin plate. The governing equation for the thin laminated plate were obtain considering the total potential energy function which was in turn minimized to obtain equation for analysis of free vibration. Work examples were resolved considering different end conditions and orientation of 0/90/90/0. The results were compared for different boundary conditions. The maximum free vibrations coefficient obtained is 5.877 and the boundary condition that gave the value was CCCC while the minimum free vibration coefficient obtained is 2.356 and the boundary condition that gave it was SSSF. From the results, it can be concluded that when the edges are free the resonating frequency of SSSS are more than 50% lower than the camped edges. Conclusively, the resonating frequency no matter the end condition decreases with increase in aspect ratio.

Index Terms— End Condition, free vibration, laminated thin plate.

1 Introduction

Vibration is the most important modes of failure in plates, it plays a crucial role in engineering [1]. The basic concepts in the mechanics of vibration are space, time, and mass (or forces). When a body is disturbed from its position, then by the elastic property of the material of the body, it tries to come back to its initial position. In general, we may see and feel that nearly everything vibrates in nature; vibrations may be sometimes very weak for identification. On the other hand, there may be large devastating vibrations that occur because of man-made disasters or natural disasters such as earthquakes, winds, and tsunamis [2]; [3]. Vibration of thin laminated rectangular composite plates is a special case of the more general problem of mechanical vibrations of thin composite plate. The free vibration of interest in rectangular laminated composite plate is the fundamental natural frequency. This is the value of externally (or naturally) induced vibrating frequency on the plate that causes it to resonate. Resonance is a dangerous phenomenon that makes a vibrating continuum to deflect excessively. Mathematically, resonance is a situation when the value of deflection of a vibrating continuum is infinity (undefined) [4]. Therefore, there exists a need for assessing the natural frequency response of structures [5].

Free vibration of thin laminated plates has been studied by some researchers using various numerical approaches. The numerical methods used so far includes Finite element method, State of art approach, Naviers and levy methods Etc [6]. The stated numerical methods have their limitations [7].

The Naviers' and levys' approaches can only be use when all the end conditions are simply supported or clamped edges or combination of simply supported and clamped edges respectively [8]; [9]. The other end conditions that have free edges have received little or no attention, the little attention given to it was by assuming a shape function which the degree of accuracy is dependent on the probability of assuming the right displacement shape function. If the assumed displacement shape function is right the results will be right, if not the reverse is the case. The finite elements methods and state of art approaches are based on either the Naviers' method or levy' method [10]. It is generally believed that finite element methods give approximate results. Due to the gap created, recently, the exact displacement method was published by Ibearugblem [11] with caption "Simple analysis of thin rectangular laminated composite with exact displacement function". In their method, Euler Bernoulli Equilibrium equation where used to formulate the exact displacement function for thin laminated composite plate considering all the forces acting on the plate. They used polynomial functional to derived the equation for each end conditions.

The present paper based its analysis on Kirchhoff's hypothesis which assumes that normal to the mid – surface of the plate before deformation remain straight and normal to the mid – surface after deformation. These theories are widely used for the analysis of thin plates [12]. Also, polynomial shape function for different plate boundary conditions will be used in this analysis. The shape function assumed that the laminated composite was made from materials that will not allow delamination of the components; that is to say that proper bonding of the

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composite must always be achieved. The displacement function for free edges were obtained from polynomial shape function published from the work of Ibearugbulem et al [4].

Euler-Bernoulli equilibrium of beam theory was applied to formulate single displacement equation for computing the free vibration of thin laminated rectangular composite plates.

The goal of this paper is to present the effect of different boundary conditions on the free vibration of thin rectangular laminated composite plate. It will also present a particular equation for obtaining the free vibration of thin rectangular laminated composite plate when pure bending and bucking are zeros.

2.0 KINEMATICS OF A LAMINA OF THIN LAMINATED PLATE

Some governing assumptions in this study is plane stress assumption (normal stress along z-axis, x-z plane and y-z plane shear stresses are zeros) another assumption is normal strain along z axis is so small that neglecting it shall not affect the gross response of the plate. Itemizing the assumptions gives:

i. $\sigma_{zz} = 0$
ii. $\tau_{xz} = 0$
iii. $\tau_{yz} = 0$
iv. $\varepsilon_{zz} = 0$

Two in-plane displacements and one out-of-plane displacement (u, v and w respectively) constitute the displacement field. From the fourth assumption it is taken that the out-of-plane displacement (deflection) is constant along z-axis, which means it is not a function of z. However, the two in-plane displacements (u and v) are functions of all coordinates (x, y and z). From assumption ii and iii, it is taken that corresponding x-z and y-z planes' shear strains are zeros. Thus, the in-plane displacements are given as shown in Equations 1 and 2:

$$u = -z\frac{dw}{dx} + u_0 \qquad (1)$$

$$v = -z\frac{dw}{dy} + v_0 \qquad (2)$$

The in-plane displacements of the middle surface (u_0 and v_0) are not constants [11].

Using Equations 1 and 2 and the no-constant values of u_0 and v_0, in-plane strains are defined as shown in Equations 3,4 and 5:

$$\varepsilon_{xx} = \frac{du}{dx} = \varepsilon_{xx}^0 + \varepsilon_{xx}^i = \frac{du_0}{dx} - z\frac{d^2w}{dx^2} \qquad (3)$$

$$\varepsilon_{yy} = \frac{dv}{dy} = \varepsilon_{yy}^0 + \varepsilon_{yy}^i = \frac{dv_0}{dy} - z\frac{d^2w}{dy^2} \qquad (4)$$

$$\gamma_{xy} = \varepsilon_{xy} + \varepsilon_{yx} = \left[-z\frac{d^2w}{dxdy} + \frac{du_0}{dy}\right] + \left[-z\frac{d^2w}{dxdy} + \frac{dv_0}{dx}\right]$$

That is:

$$\gamma_{xy} = \gamma_{xy}^0 + \gamma_{xy}^i = \frac{du_0}{dy} + \frac{dv_0}{dx} - 2z\frac{d^2w}{dxdy} \qquad (5)$$

2.1 Constitutive relations for a lamina of thin laminated plate

The Hook's law equation for one lamina in laminated plate is given as shown in Equation 6:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}e_{11} & e_{12} & 0\\ e_{12} & e_{22} & 0\\ 0 & 0 & e_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{bmatrix} \qquad (6)$$

$$Where: e_{11} = \frac{E_{11}/E_0}{1-\mu_{xy}\mu_{yx}};$$

$$e_{12} = \frac{\mu_{21}.E_{11}/E_0}{1-\mu_{xy}\mu_{yx}} = \frac{\mu_{12}.E_{22}/E_0}{1-\mu_{xy}\mu_{yx}};$$

$$e_{22} = \frac{E_{22}/E_0}{1-\mu_{xy}\mu_{yx}};\; e_{33} = \frac{G_{12}}{E_0}$$

E_0 *is the reference Elastic modulus. it can be* E_{11} *or* E_{22}. E_{ij} and μ_{ij} are the moduli of elasticity and Poisson's ratios of the anisotropic lamina. Equation 6 is transformed from the local coordinate (1-2 coordinate) using the transformation matrix [T], Equation 6 is transformed from (1-2 local) coordinate system to (x-y global) coordinate system as in Equation 7 as shown by Kubiak [13].

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy2}\end{bmatrix} = [T]^{-1}\begin{bmatrix}e_{11} & e_{12} & 0\\ e_{12} & e_{22} & 0\\ 0 & 0 & e_{33}\end{bmatrix}[T]^{-T}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} \qquad (7)$$

Where; the transformation matrix, [T] is defined as shown in Equation 8:

$$[T] = \begin{bmatrix}m^2 & n^2 & 2mn\\ n^2 & m^2 & -2mn\\ -mn & mn & (m^2-n^2)\end{bmatrix} \qquad (8)$$

$$Where: \quad m = Cos\theta \text{ and } n = Sin\theta$$

Substituting Equation 8 into Equation 7 gives Equation 9:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} \qquad (9)$$

Where:

$$a_{11} = e_{11}m^4 + 2m^2n^2(e_{12} + 2e_{33}) + e_{22}n^4$$

$$a_{12} = e_{12}m^4 + m^2n^2[e_{11} + e_{22} - 4e_{33}] + e_{12}n^4$$

$$a_{13} = m^3n[e_{11} - e_{12} - 2e_{33}] + mn^3[e_{12} - e_{22} + 2e_{33}]$$

$$a_{21} = e_{12}[m^4 + n^4] + m^2n^2[e_{11} + e_{22} - 4e_{33}]$$

$$a_{22} = e_{22}m^4 + 2m^2n^2\,[e_{12} + 2e_{33}] + e_{11}n^4$$

$$a_{23} = m^3n[e_{12} - e_{22} + 2e_{33}] + mn^3[e_{11} - e_{12} - 2f_{12}]$$

$$a_{31} = m^3n[e_{11} - e_{12} - 2e_{33}] + mn^3[e_{12} - e_{22} + 2e_{33}]$$

$$a_{32} = m^3n[e_{12} - e_{22} + 2e_{33}] + mn^3[e_{11} - e_{12} - 2e_{33}]$$

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$$a_{33} = m^2n^2[e_{11} - 2e_{12} + e_{22} - 2e_{33}] + e_{33}[m^4 + n^4]$$

Substituting Equations 3, 4 and 5 into Equation 9 gives Equation 10:

$$[\sigma] = \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} = E_0[a_{ij}][\varepsilon] \qquad 10a$$

Where:

$$[a_{ij}] = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix} \qquad 11a$$

$$[\varepsilon] = \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}\frac{du_0}{dx} - z\frac{d^2w}{dx^2}\\ \frac{dv_0}{dy} - z\frac{d^2w}{dy^2}\\ \left(\frac{du_0}{dy} + \frac{dv_0}{dx}\right) - 2z\frac{d^2w}{dxdy}\end{bmatrix} \qquad 11b$$

3.0 Total potential energy functional for a laminated thin rectangular plate

The total potential energy functional for a laminated thin rectangular plate is given as shown in Equation 12:

$$\Pi = \frac{1}{2}\iiint [\sigma][\varepsilon]\ dx\,.dy\,.dz - \frac{m\lambda^2}{2}\iint (w)^2 \quad dx\,dy \qquad 12$$

Substituting equations 10 and 11 into equation 12 gives Equation 13:

$$\Pi = \frac{E_0}{2}\iiint [\varepsilon]^T[a_{ij}][\varepsilon]\ dx\,.dy\,.dz - \frac{m\lambda^2}{2}\iint (w)^2 \quad dx\,dy \qquad 13$$

Carrying out the multiplication and closed domain integration of Equation 13 with respect to z gives Equation 14:

$$\Pi = \frac{E_0t^3}{2}\iint \Big\{ \left(\frac{A_{11}}{t^2}\left[\frac{du_0}{dx}\right]^2 + 2\frac{A_{12}}{t^2}\frac{du_0}{dx}\frac{dv_0}{dy} + \frac{A_{33}}{t^2}\left[\frac{du_0}{dy}\right]^2 + 2\frac{A_{33}}{t^2}\frac{du_0}{dy}\frac{dv_0}{dx} + \frac{A_{33}}{t^2}\left[\frac{dv_0}{dx}\right]^2 + \frac{A_{22}}{t^2}\left[\frac{dv_0}{dy}\right]^2\right)$$

$$-2\left(\frac{B_{11}}{t}\frac{du_0}{dx}.\frac{d^2w}{dx^2} + \frac{(B_{12}+2B_{33})}{t}\frac{du_0}{dy}\frac{d^2w}{dxdy} + \frac{(B_{12}+2B_{33})}{t}\frac{dv_0}{dx}\frac{d^2w}{dxdy} + \frac{B_{22}}{t}\frac{dv_0}{dy}\frac{d^2w}{dy^2}\right)$$

$$+\left(D_{11}\left[\frac{d^2w}{dx^2}\right]^2 + 2(D_{12}+2D_{33})\left[\frac{d^2w}{dxdy}\right]^2 + D_{22}\left[\frac{d^2w}{dy^2}\right]^2\right)$$

$$+2\left(\frac{A_{13}}{t^2}\frac{du_0}{dx}\frac{du_0}{dy} + \frac{A_{13}}{t^2}\frac{du_0}{dx}\frac{dv_0}{dx} - 3\frac{B_{13}}{t}\frac{du_0}{dx}\frac{d^2w}{dxdy} - \frac{B_{13}}{t}\frac{dv_0}{dx}\frac{d^2w}{dx^2} + 2D_{13}\frac{d^2w}{dx^2}\frac{d^2w}{dxdy}\right)$$

$$+2\left(\frac{A_{23}}{t^2}\frac{du_0}{dy}\frac{dv_0}{dy} + \frac{A_{23}}{t^2}\frac{dv_0}{dy}\frac{dv_0}{dx} - 3\frac{B_{23}}{t}\frac{dv_0}{dy}\frac{d^2w}{dxdy} - \frac{B_{23}}{t}\frac{du_0}{dy}\frac{d^2w}{dy^2} + 2D_{23}\frac{d^2w}{dy^2}\frac{d^2w}{dxdy}\right)\Big\}\ dx\,.dy$$

$$-\frac{m\lambda^2}{2}\iint (w)^2 \quad dx\,dy \qquad 14$$

If we assume m to stands for number of a lamina in the plate while n is the total number of laminas and:

Where:

$$A_{ij} = \frac{\overline{A_{ij}}}{t}\ \ and\ \ \overline{A_{ij}} = t\sum_{m=1}^{m=n} a_{ij}(s_m - s_{m-1}) \quad 15$$

$$B_{ij} = \frac{\overline{B_{ij}}}{t^2}\ and\ \overline{B_{ij}} = \frac{t^2}{2}\sum_{m=1}^{m=n} a_{ij}(s_m^2 - s_{m-1}^2) \quad 16$$

$$D_{ij} = \frac{\overline{D_{ij}}}{t^3}\ and\ \overline{D_{ij}}. = \frac{t^3}{3}\sum_{m=1}^{m=n} a_{ij}(s_m^3 - s_{m-1}^3) \quad 17$$

"m" stands for the number of a lamina in the laminated plate, n is the total number of laminas "s" is the non dimensional coordinate along z-axis defined as s = z/t.

Let the summation of the following three constants be one. That is:

$$n_1 + n_2 + n_3 = 1 \qquad (18)$$

Substituting Equation 18 into Equation 14 to multiply the free vibration coefficient, $m\lambda^2$ (that is: $m\lambda^2 = n_1 m\lambda^2 + n_2 m\lambda^2 + n_3 m\lambda^2$) and rearranging the resulting gives Equation 19

$$\Pi = \Pi_1 + \Pi_2 + \Pi_3 \qquad (19)$$

$$\Pi_1 = \frac{E_0t^3}{2}\iint \Big\{ \left(\frac{A_{11}}{t^2}\left[\frac{du_0}{dx}\right]^2 + 2\frac{A_{12}}{t^2}\frac{du_0}{dx}\frac{dv_0}{dy} + \frac{A_{33}}{t^2}\left[\frac{du_0}{dy}\right]^2 + 2\frac{A_{33}}{t^2}\frac{du_0}{dy}\frac{dv_0}{dx} + \frac{A_{33}}{t^2}\left[\frac{dv_0}{dx}\right]^2 + \frac{A_{22}}{t^2}\left[\frac{dv_0}{dy}\right]^2\right)$$

$$-2\left(\frac{B_{11}}{t}\frac{du_0}{dx}.\frac{d^2w}{dx^2} + \frac{(B_{12}+2B_{33})}{t}\frac{du_0}{dy}\frac{d^2w}{dxdy} + \frac{(B_{12}+2B_{33})}{t}\frac{dv_0}{dx}\frac{d^2w}{dxdy} + \frac{B_{22}}{t}\frac{dv_0}{dy}\frac{d^2w}{dy^2}\right)$$

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$$+\left(D_{11}\left[\frac{d^2w}{dx^2}\right]^2+2(D_{12}+2D_{33})\left[\frac{d^2w}{dxdy}\right]^2+D_{22}\left[\frac{d^2w}{dy^2}\right]^2\right)-n_1\frac{m\lambda^2}{2}\iint(w)^2\quad dx\,dy \qquad 20a$$

$$\Pi_2=\frac{2E_0t^3}{2}\iint\left[\frac{A_{13}}{t^2}\frac{du_0}{dx}\frac{du_0}{dy}+\frac{A_{13}}{t^2}\frac{du_0}{dx}\frac{dv_0}{dx}-3\frac{B_{13}}{t}\frac{du_0}{dx}\frac{d^2w}{dxdy}-\frac{B_{13}}{t}\frac{dv_0}{dx}\frac{d^2w}{dx^2}+2D_{13}\frac{d^2w}{dx^2}\frac{d^2w}{dxdy}\right]dx\,.dy\;-n_2\frac{m\lambda^2}{2}\iint(w)^2\quad dx\,dy \quad 20b$$

$$\Pi_3=\frac{2E_0t^3}{2}\iint\left[\frac{A_{23}}{t^2}\frac{du_0}{dy}\frac{dv_0}{dy}+\frac{A_{23}}{t^2}\frac{dv_0}{dy}\frac{dv_0}{dx}-3\frac{B_{23}}{t}\frac{dv_0}{dy}\frac{d^2w}{dxdy}-\frac{B_{23}}{t}\frac{du_0}{dy}\frac{d^2w}{dy^2}+2D_{23}\frac{d^2w}{dy^2}\frac{d^2w}{dxdy}\right]dx\,.dy-n_3\frac{m\lambda^2}{2}\iint(w)^2\quad dx\,dy \qquad 20c$$

The meaning for z, m and n for easy understanding is illustrated with the laminated plate of four laminas that is shown on Figure 1

$z_0 = t/2$

Lamina 1: $m = 1;\ z_{m-1} = z_0;\ z_m = z_1$

$z_1 = t/4$

Lamina 2: $m = 2;\ z_{m-1} = z_1;\ z_m = z_2$

$z_2 = 0$

Lamina 3: $m = 3;\ z_{m-1} = z_2;\ z_m = z_3$

$z_3 = -t/4$

Lamina 4: $m = 4;\ z_{m-1} = z_3;\ z_m = z_4$

$z_4 = -t/2$

Figure 1: A laminated plate that is made of four laminas

3.1 General and direct Variation of Total potential energy functional for a laminated thin rectangular plate

Minimizing Equations 20a, 20b and 20c with respect to w, u_0 and v_0 and making some rearrangements shall give the respective equations:

$$\frac{\partial\Pi_1}{\partial w}=0=\iint\left[-\frac{1}{t}\left(B_{11}\frac{\partial^3u_0}{\partial x^3}+(B_{12}+2B_{33})\frac{\partial^3u_0}{\partial x\partial y^2}+(B_{12}+2B_{33})\frac{\partial^3v_0}{\partial x^2\partial y}+B_{22}\frac{\partial^3v_0}{\partial y^3}\right)+\left(D_{11}\frac{\partial^4w}{\partial x^4}+2(D_{12}+2D_{33})\frac{\partial^4w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4w}{\partial y^4}\right)\right]dxdy+\frac{n_1m\lambda^2}{E_0t^3}\iint w\,dx\,dy \qquad 21a$$

$$\frac{\partial\Pi_2}{\partial w}=0=\frac{1}{t}\iint\frac{\partial^2}{\partial x^2}\left(-3B_{13}\frac{\partial u_0}{\partial y}-B_{13}\frac{\partial v_0}{\partial x}+4D_{13}\frac{\partial^2w}{\partial x\partial y}\right)\;dxdy+\frac{n_2m\lambda^2}{E_0t^3}\iint w\,dx\,dy \qquad 21b$$

$$\frac{\partial\Pi_3}{\partial w}=0=\frac{1}{t}\iint\frac{\partial^2}{\partial y^2}\left(-3B_{23}\frac{\partial v_0}{\partial x}-B_{23}\frac{\partial u_0}{\partial y}+4D_{23}\frac{\partial^2w}{\partial x\partial y}\right)\;dxdy+\frac{n_3m\lambda^2}{E_0t^3}\iint w\,dx\,dy \qquad 21c$$

$$\frac{\partial\Pi_1}{\partial u_0}=0=\frac{1}{t^2}\iint\left(\frac{d^2}{dx^2}\left[A_{11}u_0-B_{11}\frac{dw}{dx}\right]+\frac{d^2}{dxdy}\left[A_{12}v_0-B_{12}\frac{dw}{dy}\right]+\frac{d^2}{dxdy}\left[A_{33}v_0-B_{33}\frac{dw}{dy}\right]+\frac{d^2u_0}{dy^2}\left[A_{33}u_0-B_{33}\frac{dw}{dx}\right]\right)dxdy \quad 22a$$

$$\frac{\partial\Pi_2}{\partial u_0}=\iint\left(2\frac{d^2}{dxdy}\left[\frac{A_{13}}{t^2}u_0-\frac{B_{13}}{t}\frac{dw}{dx}\right]+\frac{d^2}{dx^2}\left[\frac{A_{13}}{t^2}v_0-\frac{B_{13}}{t}\frac{dw}{dy}\right]\right)\;dxdy=0 \qquad 22b$$

$$\frac{\partial\Pi_3}{\partial u_0}=\iint\frac{d^2}{dy^2}\left[\frac{A_{23}}{t^2}v_0-\frac{B_{23}}{t}\frac{du_w}{dy}\right]\;dxdy=0 \qquad 22c$$

$$\frac{\partial\Pi_1}{\partial v_0}=\iint\left[\frac{d^2}{dxdy}\left(\left[\frac{A_{12}}{t^2}u_0-\frac{B_{12}}{t}\frac{dw}{dx}\right]+\frac{d^2}{dy^2}\left[\frac{A_{22}}{t^2}v_0-\frac{B_{22}}{t}\frac{dw}{dy}\right]+\frac{d^2}{dxdy}\left[\frac{A_{33}}{t^2}u_0-\frac{B_{33}}{t}\frac{dw}{dx}\right]+\frac{d^2}{dx^2}\left[\frac{A_{33}}{t^2}v_0-\frac{B_{33}}{t}\frac{dw}{dy}\right]\right)\right]dxdy\;=0 \quad 23a$$

$$\frac{\partial\Pi_2}{\partial v_0}=\iint\frac{d^2}{dx^2}\left[\frac{A_{13}}{t^2}u_0-\frac{B_{13}}{t}\frac{dw}{dx}\right]\;dxdy=0 \qquad 23b$$

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$$\frac{\partial \Pi_3}{\partial v_0} = \iint \left(\frac{d^2}{dy^2}\left[\frac{A_{23}}{t^2} u_0 - \frac{B_{23}}{t}\frac{dw}{dx} \right] + 2\frac{d^2}{dxdy}\left[\frac{A_{23}}{t^2} v_0 - \frac{B_{23}}{t}\frac{dw}{dy} \right] \right) dxdy = 0 \quad 23c$$

For Equations 22a, 22b, 22c, 23a, 23b and 23c to be true, the following shall hold (where c and d are yet to be determined constants)):

$$u_0 = t\frac{B_{ij}}{A_{ij}}\frac{\partial w}{\partial x} = c.t\frac{\partial w}{\partial x} \quad 24a$$

$$v_0 = t\frac{B_{ij}}{A_{ij}}\frac{\partial w}{\partial y} = d.t\frac{\partial w}{\partial y} \quad 24b$$

Substituting Equations 24a and 24b into equation 21a and making some rearrangements and observing that an integral can only be zero if its integrand is as shown in Equation 25:

$$\iint \left([D_{11} - cB_{11}]\frac{\partial^4 w}{\partial x^4} + 2[D_{12} - cB_{12} - dB_{12} + 2D_{33} - 2cB_{33} - 2dB_{33}]\frac{\partial^4 w}{\partial x^2 \partial y^2} + [D_{22} - dB_{22}]\frac{\partial^4 w}{\partial y^4} + \frac{n_1 m\lambda^2}{E_0 t^3}.w \right) dxdy = 0 \quad 25$$

Dividing equation 25 by [D22 - dB22] gives Equation 26:

$$\iint \left[f_1 \frac{\partial^4 w}{\partial x^4} + f_2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{n_1 m\lambda^2}{E_0 t^3}.w \right] dxdy = 0 \quad 26$$

$$Where:\ f_1 = \frac{[D_{11} - cB_{11}]}{[D_{22} - dB_{22}]};\ f_2 = \frac{2[D_{12} - cB_{12} - dB_{12} + 2D_{33} - 2cB_{33} - 2dB_{33}]}{[D_{22} - dB_{22}]}$$

The exact solution to Equation 26 is in polynomial form (in terms of non-dimensional coordinates) (see [14]] for details):

$$w = \left(a_0 + a_1 x + \frac{a_2}{2!}x^2 + \frac{a_3}{3!}x^3 + \frac{a_4}{4!}x^4 \right)\left(b_0 + b_1 y + \frac{b_2}{2!}y^2 + \frac{b_3}{3!}y^3 + \frac{b_4}{4!}y^4 \right) \quad 27a$$

From Equations 27a, it was gathered that:

$$w = \beta_1 h \quad 27b$$

Substituting Equation 27b into Equations 24a and 24b gives:

$$u_0 = c.t.\beta_1 \frac{\partial h}{\partial x} = \beta_2 \frac{\partial h}{\partial x} \quad 28a$$

$$v_0 = d.t.\beta_1 \frac{\partial h}{\partial y} = \beta_3 \frac{\partial h}{\partial y} \quad 28b$$

$$\beta_2 = c.t.\beta_1 \ \ and \ \ \beta_3 = d.t.\beta_1 \quad 28c$$

Substituting Equations 27b, 28a and 28b into Equations 20a, 20b and 20c and writing the outcomes in terms of non dimensional coordinates gives:

$$\Pi_1 = \frac{E_0 t^3 ab}{2a^4}\iint \left[\left(A_{11}\beta_2^2 \left(\frac{\partial^2 h}{\partial R^2}\right)^2 + \frac{1}{\alpha^2}[2A_{12}\beta_2\beta_3 + 2A_{33}\beta_2\beta_3 + A_{33}\beta_2^2 + A_{33}\beta_3^2]\left(\frac{\partial^2 h}{\partial R \partial Q}\right)^2 + A_{22}\frac{\beta_3^2}{\alpha^4}\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 \right) - 2\left(B_{11}\beta_1\beta_2 \left(\frac{\partial^2 h}{\partial R^2}\right)^2 + (B_{12} + 2B_{33})\frac{\beta_1\beta_2}{\alpha^2}\left(\frac{\partial^2 h}{\partial R \partial Q}\right)^2 + (B_{12} + 2B_{33})\frac{\beta_1\beta_3}{\alpha^2}\left(\frac{\partial^2 h}{\partial R \partial Q}\right)^2 + B_{22}\frac{\beta_1\beta_3}{\alpha^4}\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 \right) + \left(D_{11}\beta_1^2 \left(\frac{\partial^2 h}{\partial R^2}\right)^2 + 2(D_{12} + 2D_{33})\frac{\beta_1^2}{\alpha^2}\left(\frac{\partial^2 h}{\partial R \partial Q}\right)^2 + D_{22}\frac{\beta_1^2}{\alpha^4}\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 \right) \right] dRdQ$$

$$-\frac{n_1 m\lambda^2 ab}{2}\beta_1^2 \iint (h)^2\, dR\, dQ \quad 29a$$

$$\Pi_2 = \frac{2E_0 t^3 ab}{2a^4}\iint \left[A_{13}\frac{\beta_2^2}{\alpha}\frac{\partial^2 h}{\partial R^2}.\frac{\partial^2 h}{\partial R \partial Q} + A_{13}\frac{\beta_2\beta_3}{\alpha}\frac{\partial^2 h}{\partial R^2}.\frac{\partial^2 h}{\partial R \partial Q} - 3B_{13}\frac{\beta_1\beta_2}{\alpha}\frac{\partial^2 h}{\partial R^2}\frac{\partial^2 h}{\partial R \partial Q} - B_{13}\frac{\beta_1\beta_3}{\alpha}.\frac{\partial^2 h}{\partial R^2}\frac{\partial^2 h}{\partial R \partial Q} + 2D_{13}\frac{\beta_1^2}{\alpha}\frac{\partial^2 h}{\partial R^2}\frac{\partial^2 h}{\partial R \partial Q} \right] dR\,.dQ - \frac{n_2 m\lambda^2 ab}{2}\beta_1^2 \iint (h)^2\, dR\, dQ \quad 29b$$

$$\Pi_3 = \frac{2E_0 t^3 ab}{2a^4}\iint \left[A_{23}\frac{\beta_2\beta_3}{\alpha^3}\frac{\partial^2 h}{\partial R \partial Q}.\frac{\partial^2 h}{\partial Q^2} + A_{23}\frac{\beta_3^2}{\alpha^3}\frac{\partial^2 h}{\partial R \partial Q}.\frac{\partial^2 h}{\partial Q^2} - 3B_{23}\frac{\beta_1\beta_3}{\alpha^3}\frac{\partial^2 h}{\partial R \partial Q}.\frac{\partial^2 h}{\partial Q^2} - B_{23}\frac{\beta_1\beta_2}{\alpha^3}\frac{\partial^2 h}{\partial R \partial Q}.\frac{\partial^2 h}{\partial Q^2} + 2D_{23}\frac{\beta_1^2}{\alpha^3}\frac{\partial^2 h}{\partial R \partial Q}.\frac{\partial^2 h}{\partial Q^2} \right] dR\,.dQ - \frac{n_3 m\lambda^2 ab}{2}\beta_1^2 \iint (h)^2\, dR\, dQ \quad 29c$$

Minimizing Equations 29a, 29b and 29c with respect to β1 and rearrange gives respectively Equation 30:

$$\frac{d\Pi_1}{d\beta_1} = 0 = -\left(B_{11}\beta_2 k_x + (B_{12} + 2B_{33})\frac{\beta_2}{\alpha^2}k_{xy} + (B_{12} + 2B_{33})\frac{\beta_3}{\alpha^2}k_{xy} + B_{22}\frac{\beta_3}{\alpha^4}k_y \right) + \beta_1\left(D_{11}k_x + \frac{2}{\alpha^2}(D_{12} + 2D_{33})k_{xy} + \frac{D_{22}}{\alpha^4}k_y \right) - \frac{n_1 m\lambda^2}{E_0 t^3}\beta_1 k_\lambda \quad 30a$$

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$$\frac{d\Pi_2}{d\beta_1} = 0 = \left(4D_{13}\frac{\beta_1}{\alpha} - 3B_{13}\frac{\beta_2}{\alpha} - B_{13}\frac{\beta_3}{\alpha}\right)k_{xxy} - \frac{n_2 m\lambda^2}{E_0 t^3}\beta_1 k_\lambda \quad 30b$$

$$\frac{d\Pi_3}{d\beta_1} = 0 = \left(4D_{23}\frac{\beta_1}{\alpha^3} - B_{23}\frac{\beta_2}{\alpha^3} - 3B_{23}\frac{\beta_3}{\alpha^3}\right)k_{xyy} - \frac{n_3 m\lambda^2}{E_0 t^3}\beta_1 k_\lambda \quad 30c$$

Minimizing Equations 29a, 29b and 29c with respect to β2 gives Equation 31:

$$\frac{d\Pi_1}{d\beta_2} = \frac{ab}{\alpha^4}\left[\left(A_{11}\beta_2 k_x + \frac{1}{\alpha^2}[A_{12}\beta_3 + A_{33}\beta_2 + A_{33}\beta_3]k_{xy}\right) - B_{11}\beta_1 k_x - (B_{12} + 2B_{33})\frac{\beta_1}{\alpha^2}k_{xy}\right] = 0 \quad 31a$$

$$\frac{d\Pi_2}{d\beta_2} = \frac{ab}{\alpha^4}\left[2A_{13}\frac{\beta_2}{\alpha}k_{xxy} + A_{13}\frac{\beta_3}{\alpha}k_{xxy} - 3B_{13}\frac{\beta_1}{\alpha}k_{xxy}\right] = 0 \quad 31b$$

$$\frac{d\Pi_3}{d\beta_2} = 0 = \frac{ab}{\alpha^4}\left[A_{23}\frac{\beta_3}{\alpha^3}k_{xyy} - B_{23}\frac{\beta_1}{\alpha^3}k_{xyy}\right] \quad 31c$$

Minimizing Equations 29a, 29b and 29c with respect to β_3 gives Equation 32:

$$\frac{d\Pi_1}{d\beta_3} = \frac{ab}{\alpha^4}\left[A_{22}\frac{\beta_3}{\alpha^4}k_y + \frac{1}{\alpha^2}[A_{12}\beta_2 + A_{33}\beta_2 + A_{33}\beta_3]k_{xy} - B_{22}\frac{\beta_1}{\alpha^4}k_y - (B_{12} + 2B_{33})\frac{\beta_1}{\alpha^2}k_{xy}\right] = 0 \quad 32a$$

$$\frac{d\Pi_2}{d\beta_3} = 0 = \frac{ab}{\alpha^4}\left[A_{13}\frac{\beta_2}{\alpha}k_{xxy} - B_{13}\frac{\beta_1}{\alpha}.k_{xxy}\right] \quad 32b$$

$$\frac{d\Pi_3}{d\beta_3} = 0 = \frac{ab}{\alpha^4}\left[A_{23}\frac{\beta_2}{\alpha^3}k_{xyy} + 2A_{23}\frac{\beta_3}{\alpha^3}k_{xyy} - 3B_{23}\frac{\beta_1}{\alpha^3}k_{xyy}\right] \quad 32c$$

Where stiffness coefficients,

$$k_x = \iint\left(\frac{\partial^2 h}{\partial R^2}\right)^2 dR\,.dQ \;:\; k_{xy} = \iint\left(\frac{\partial^2 h}{\partial R\partial Q}\right)^2 dR\,.dQ \;:\; k_y = \iint\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 dR\,.dQ$$

$$k_{xxy} = \iint \frac{\partial^2 h}{\partial R^2}\frac{\partial^2 h}{\partial R\partial Q}\, dR\,.dQ : k_{xyy} = \iint \frac{\partial^2 h}{\partial R\partial Q}.\frac{\partial^2 h}{\partial Q^2}\, dR\,.dQ \;:\; k_q = \iint h\, dR\,.dQ$$

$$k_N = \iint\left(\frac{dh}{dR}\right)^2 dR\,.dQ : k_\lambda = \iint h^2\, dR\,.dQ$$

Adding the Equations 30a, 30b and 30c together and rearranging the outcome gives:

$$\frac{d\Pi}{d\beta_1} = \frac{d\Pi_1}{d\beta_1} + \frac{d\Pi_2}{d\beta_1} + \frac{d\Pi_3}{d\beta_1} = 0. \quad \textit{That is:}$$

$$\frac{d\Pi}{d\beta_1} = \beta_1\left(D_{11}k_x + \frac{2}{\alpha^2}(D_{12} + 2D_{33})k_{xy} + \frac{D_{22}}{\alpha^4}k_y + 4\frac{D_{13}}{\alpha}k_{xxy} + 4\frac{D_{23}}{\alpha^3}k_{xyy}\right)$$

$$-\beta_2\left(B_{11}k_x + (B_{12} + 2B_{33})\frac{k_{xy}}{\alpha^2} + 3B_{13}\frac{k_{xxy}}{\alpha} + B_{23}\frac{k_{xyy}}{\alpha^3}\right)$$

$$-\beta_3\left((B_{12} + 2B_{33})\frac{k_{xy}}{\alpha^2} + B_{22}\frac{k_y}{\alpha^4} + B_{13}\frac{k_{xxy}}{\alpha} + 3B_{23}\frac{k_{xyy}}{\alpha^3}\right) - \frac{\alpha^4}{E_0 t^3}(n_1 + n_2 + n_3)m\lambda^2\beta_1 k_\lambda \quad 33a$$

Substituting Equation 18 into Equation 33a and rearranging the outcome gives Equation 33b:

$$\frac{m\lambda^2\alpha^4}{E_0 t^3}\beta_1 k_\lambda = \beta_1\left(D_{11}k_x + \frac{2}{\alpha^2}(D_{12} + 2D_{33})k_{xy} + \frac{D_{22}}{\alpha^4}k_y + 4\frac{D_{13}}{\alpha}k_{xxy} + 4\frac{D_{23}}{\alpha^3}k_{xyy}\right)$$

$$-\beta_2\left(B_{11}k_x + (B_{12} + 2B_{33})\frac{k_{xy}}{\alpha^2} + 3B_{13}\frac{k_{xxy}}{\alpha} + B_{23}\frac{k_{xyy}}{\alpha^3}\right)$$

$$-\beta_3\left((B_{12} + 2B_{33})\frac{k_{xy}}{\alpha^2} + B_{22}\frac{k_y}{\alpha^4} + B_{13}\frac{k_{xxy}}{\alpha} + 3B_{23}\frac{k_{xyy}}{\alpha^3}\right) \quad 33b$$

Adding the Equations 31a, 31b and 31c together and rearranging the outcome gives Equation 34:

$$\beta_2\left(A_{12}\frac{k_{xy}}{\alpha^2} + A_{33}\frac{k_{xy}}{\alpha^2} + A_{13}\frac{k_{xxy}}{\alpha} + A_{23}\frac{k_{xyy}}{\alpha^3}\right) + \beta_3\left(A_{22}\frac{k_y}{\alpha^4} + A_{33}\frac{k_{xy}}{\alpha^2} + 2A_{23}\frac{k_{xyy}}{\alpha^3}\right) = \beta_1\left(B_{22}\frac{k_y}{\alpha^4} + (B_{12} + 2B_{33})\frac{k_{xy}}{\alpha^2} + B_{13}\frac{k_{xxy}}{\alpha} + 3B_{23}\frac{k_{xyy}}{\alpha^3}\right) \quad 34$$

Solving Equations 33b and 34 simultaneously gives Equation 35:

$$\beta_2 = T_2\beta_1 = \beta_1\frac{(d_{12}.d_{23} - d_{13}.d_{22})}{({d_{12}}^2 - d_{11}d_{22})} \quad 35a$$

$$\beta_3 = T_3\beta_1 = \beta_1\frac{(d_{12}.d_{13} - d_{11}d_{23})}{({d_{12}}^2 - d_{11}d_{22})} \quad 35b$$

Where:

$$d_{11} = A_{11}k_x + A_{33}\frac{k_{xy}}{\alpha^2} + 2A_{13}\frac{k_{xxy}}{\alpha} \quad 36a$$

$$d_{12} = A_{12}\frac{k_{xy}}{\alpha^2} + A_{33}\frac{k_{xy}}{\alpha^2} + A_{13}\frac{k_{xxy}}{\alpha} + A_{23}\frac{k_{xyy}}{\alpha^3} \quad 36b$$

$$d_{22} = A_{22}\frac{k_y}{\alpha^4} + A_{33}\frac{k_{xy}}{\alpha^2} + 2A_{23}\frac{k_{xyy}}{\alpha^3} \quad 36c$$

$$d_{13} = B_{11}k_x + (B_{12} + 2B_{33})\frac{k_{xy}}{\alpha^2} + 3B_{13}\frac{k_{xxy}}{\alpha} + B_{23}\frac{k_{xyy}}{\alpha^3} \quad 36d$$

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$$d_{23} = B_{22}\frac{k_y}{\alpha^4} + (B_{12} + 2B_{33})\frac{k_{xy}}{\alpha^2} + B_{13}\frac{k_{xxy}}{\alpha} + 3B_{23}\frac{k_{xyy}}{\alpha^3} \quad 36e$$

Substituting Equations 35a and 35b into Equation 33b and rearranging gives:

$$\frac{m\lambda^2 a^4}{D_0 t^3}k_\lambda = E_0\left\{\left(\frac{D_{11}}{D_0}k_x + \frac{2}{\alpha^2}\left(\frac{D_{12}}{D_0} + 2\frac{D_{33}}{D_0}\right)k_{xy} + \frac{D_{22}}{D_0}.\frac{k_y}{\alpha^4} + 4\frac{D_{13}}{D_{22}}.\frac{k_{xxy}}{\alpha} + 4\frac{D_{23}}{D_{22}\alpha^3}k_{xyy}\right) - \frac{T_2}{D_0}\left(B_{11}k_x + \frac{(B_{12}+2B_{33})}{\alpha^2}k_{xy} + \frac{3B_{13}}{\alpha}k_{xxy} + \frac{B_{23}}{\alpha^3}k_{xyy}\right) - \frac{T_3}{D_0}\left(\frac{(B_{12}+2B_{33})}{\alpha^2}k_{xy} + \frac{B_{22}}{\alpha^4}k_y + \frac{B_{13}}{\alpha}k_{xxy} + \frac{3B_{23}}{\alpha^3}k_{xyy}\right)\right\} \quad 37$$

Rearranging Equation 37 gives Equation 38:

$$\frac{m\lambda^2 a^4}{D_0 t^3} = \frac{m\lambda^2 a^4}{\overline{D_0}} = E_0\left(\frac{k_{T1} + k_{T2} + k_{T3}}{k_\lambda}\right) \quad 38$$

Equation 38 is the exact displacement Equation for computing the free vibration of thin rectangular laminated composite plates

Where:

$$k_{T1} = \left(\frac{D_{11}}{D_0}k_x + \frac{2}{\alpha^2}\left(\frac{D_{12}}{D_0} + 2\frac{D_{33}}{D_0}\right)k_{xy} + \frac{D_{22}}{D_0}.\frac{k_y}{\alpha^4} + 4\frac{D_{13}}{D_{22}}.\frac{k_{xxy}}{\alpha} + 4\frac{D_{23}}{D_{22}\alpha^3}k_{xyy}\right) \quad 40a$$

$$k_{T2} = -\frac{T_2}{D_0}\left(B_{11}k_x + \frac{(B_{12}+2B_{33})}{\alpha^2}k_{xy} + \frac{3B_{13}}{\alpha}k_{xxy} + \frac{B_{23}}{\alpha^3}k_{xyy}\right) \quad 40b$$

$$k_{T3} = -\frac{T_3}{D_0}\left(\frac{(B_{12}+2B_{33})}{\alpha^2}k_{xy} + \frac{B_{22}}{\alpha^4}k_y + \frac{B_{13}}{\alpha}k_{xxy} + \frac{3B_{23}}{\alpha^3}k_{xyy}\right) \quad 40b$$

4.0 Numerical examples

A thin rectangular plate with orientation 0/90/90/0 having material properties as follows: - $G_{12}/E_2 = 0.5$; $V_{12} = 0.25$; $E_1/E_2 = 25$, aspect ratio ranging from 1 to 2. It is required to determine fundamental natural frequency when the plate is undergoing free vibration considering the following end conditions SSSS, CCCC, CSCS, CSSS, SSSF and CCCF. SSSS represents a rectangular composite plate with all edges simply supported, CCCC represent a rectangular composite plate with all edges clamped, CSCS represents a rectangular composite plate with two opposite edges simply supported while the other two are clamped, CSSS, represents a rectangular composite plate with all the edges simply supported except the first edge that is clamped. SSSF represents a rectangular composite plate with all three edges simply supported while the fourth edge is free; CCCF represents a rectangular composite plate with two adjacent edges clamped while the other two are free. The reference elastic modulus, E_0 is taken to be E_2. Hence,

$$\frac{a^4 m\lambda^2}{D_0 t^3} = \frac{a^4 m\lambda^2}{\overline{D_0}} = \frac{a^4 m\lambda^2}{\overline{D_2}} = E_2\left(\frac{k_{T1} + k_{T2} + k_{T3}}{k_\lambda}\right) \quad 41$$

If the aspect ratio is a/b and the parameters are in terms of long length "b" then:

$$m\lambda^2 b^4 = [m\lambda^2 a^4] \times [b/a]^4$$

The fundamental natural frequency analyses of SSSS, CCCC, CSCS, CSSS, SSSF and CCCF plates after satisfying the boundary condition using equations 27b are as shown in Table 1 using polynomial functions while considering the numbering pattern shown in figure 1.

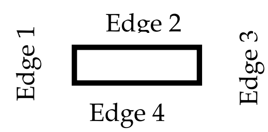

Figure 1: Numbering pattern of the plate's edges.

Table 1: Plate displacement equation considering polynomial function

Plates End conditions	Deflection equation
SSSS plate	$W = \beta_1 h = \beta_1(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$
CCCC plate	$W = \beta_1 h = \beta_1(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$
CSCS plate	$W = \beta_1 h = \beta_1(R^2 - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$
CSSS Plate	$W = \beta_1 h = \beta_1(1.5R^2 - 2.5R^3 + R^4)(Q - 2Q^3 + Q^4)$
CCCF plate	$W = \beta_1 h = \beta_1(R^2 - 2R^3 + R^4)(2.8Q^2 - 5.2Q^3 + 3.8Q^4 - Q^5)$
SSSF plate	$W = C_1(R - 2R^3 + R^4)\left(\frac{7Q}{3} - \frac{10Q^3}{3} + \frac{10Q^4}{3} - Q^5\right)$

The stiffness coefficients obtained by considering the displacement function shown in table 1 are presented in table 2

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Table 2: Stiffness coefficients (k-vakues) for different plate boundary conditions

PLATE TYPE	K_x	K_{xy}	K_y	K_{xxy}	K_{xyy}	K_λ
SSSS	0.2362	0.2359	0.2362	0	0	0.02421
CCCC	0.00127	0.000361	0.00127	0	0	0.0000252
CSCS	0.039365	0.009233	0.007619	0	0	0.0000781
CSSS	0.088571	0.041629	0.036192	0	0	0.000371
SSSF	4.025763	1.033074	0.187451	0	0	0.041269
CCCF	0.048463	0.0035983	0.004151	0	0	0.000095921

4.1 Results and Discussions

Substituting the values of stiffness coefficient and material properties given in the question to Equation 41 yields results presented on Table 3 in terms of long length of the plate.

From Table 3 and Figure 2 one will observed that with constant E_1/E_2, increases in aspect ratio will cause decrease in the value of resonating frequencies no matter the boundary condition under consideration. It can also be seen that that the percentage difference between the CCCC Plates and SSSS PLATE is more than 50%. The maximum resonating frequency occurs when the aspect ratio (a/b) is equal to unity (square plate) with maximum coefficient value of 5.877, when the aspect ratio (a/b) is equal to 2 the minimum resonating frequency coefficient value is equal to 2.356. The boundary condition that produces the least resonating frequency is SSSF

Table 3: Natural frequencies for different boundary conditions in terms of long length "a" from the present study, $\left(\sqrt{\frac{D_{22}}{a^4 m}}\right) \div \Pi^2$ for aspect ratio of 1 to 2.

Orientation = 0/90/90/0 $G_{12}/E_2 = 0.5$; $\mu_{12} = 0.25$, $E_1/E_2 = 25$						
$\propto = a/b$	SSSS	CCCC	CSCS	CSSS	SSSF	CCCF
1	2.671051031	5.877415404	5.497061633	2.945760301	2.390573623	5.404786492
1.1	2.590119836	5.722346299	5.456083289	2.788531617	2.381646804	5.387875475
1.2	2.534811574	5.620125956	5.428430526	2.681235566	2.375230705	5.376644253
1.3	2.495762559	5.550519087	5.409045097	2.606029318	2.370462636	5.368885221
1.4	2.467388213	5.501726554	5.395007856	2.551996037	2.366820425	5.3633427
1.5	2.446240681	5.466630316	5.384557356	2.51228465	2.363973502	5.359268518
1.6	2.430122619	5.440798811	5.376588688	2.4824894	2.361704485	5.356198699
1.7	2.417592496	5.421392169	5.370384849	2.459710215	2.359865699	5.353835323
1.8	2.407679401	5.406541986	5.36546645	2.441995554	2.35835397	5.351981176
1.9	2.399713601	5.394989227	5.361504383	2.428004701	2.357095431	5.350502144
2.0	2.393223002	5.385866829	5.358267331	2.416798626	2.356036051	5.349304817

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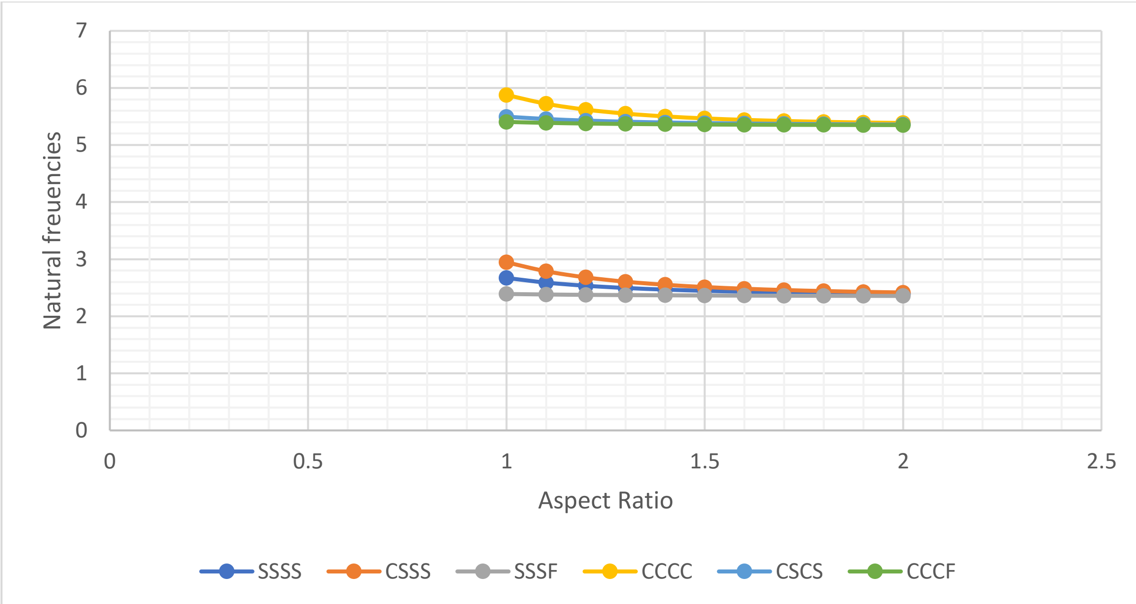

Figure 2: Graph of natural frequencies against aspect ratio for different boundary conditions.

5.0 Conclusion

The resonating frequency is minimum when the thin laminated composite plate has three simply supported edges with one free edge, the aspect ratio these occur was two (2). The maximum resonating frequency occurs when all the edges are clamped with aspect ratio of one. This is because of the rigidity of the clamped boundary condition.

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